TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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GRAPHIC METHOD FOR CALCULATING THE SPEED AND CLIMBING ABILITY OF AIRPLANES.

By Adolf Rohrbach and Edwin Lupberger.
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GRAPHIC METHOD FOR CALCULATING THE SPEED AND CLIMBING ABILITY OF AIRPLANES.*

By Adolf Rohrbach and Edwin Lupberger.

The simplest known methods for calculating airplane performances are those of Everling (Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1916, p.125), and Kann (Technische Berichte, Vol.I, No.6, page 231). Both methods have this feature in common, that they express the particular quantity which is to be determined (time of climb, speed, etc.) by a single formula. Thus, by using Everling's airplane calculation tables, we are able to find related values (e.g. gliding coefficient $K = \frac{c_a}{c_w}$, ceiling, and total weight per HP), by a very simple graphical method. In order to obtain accurate results, however, the assumptions upon which these formulas are based should be constantly borne in mind. They assume, for example, that certain definite relations exist between the engine output and the air density. But whenever it was desired to utilize the results of brake tests on airplane engines carried out in the low-pressure chamber of the Zeppelin airship works at Friedrichshafen (Technische Berichte, Vol.III, No.1, p.1), in order to calculate the times of climb, etc., separate formulas had to be established for each type of engine, for calculating the flight performances of the airplane.

*From Technische Berichte, Volume III, No.6, pp. 218-221. (1918).

A new method is, therefore, given here, by means of which, starting from the actual power developed by the engine, it is possible to determine all the factors which together constitute what are known as the flight performances (such as horizontal speed, climbing speed at various angles of attack and at different altitudes, the ceiling, etc.) for any combination of engine and airplane, from the single graphic diagram given herewith. This method is a development of the one given by Prof. A. Baumann (Mechanische Grundlagen des Flugzeugbaues, Vol.I, p.145), according to which the drag, the propeller thrust and speed of an airplane are considered dependent on the angle of attack.

We shall first describe the calculations necessary for determining flight performances of biplanes fitted with the supercompressed Maybach engine type MbIVa. In the first place, it is necessary to know the curve of polar characteristics for the entire airplane, which can be determined best by experiments with models, though it may also be obtained by calculation on the basis of extensive published data (Technische Berichte, Vol. I, No.4, pp.85, 98 and 103; No.5, p.148; No.6, pp.204 et. seq.). This polar curve is then plotted in the top right-hand square, at 3° intervals of the angle of attack, for which purpose the scale of drag coefficients c_w is given vertically on the right, and that of the lift coefficients c_a underneath toward the left.

The biplane in the example selected is assumed to have a wing loading of 40 kg/sq.m. The airspeed required to maintain horizontal flight at an altitude of 3000 meters when flying at an angle of attack of 1.5° is found in the following manner: From the point on the polar curve corresponding to an angle of attack of 1.5°, a "transfer" line is drawn vertically downward, in the direction indicated by the arrow, till it cuts the curve of wing loading marked 40 kg/sq.m. From this point of intersection, it is carried horizontally toward the left to the line corresponding to an altitude of 3000 meters. At the point where it cuts the 3000-meter line, a vertical is drawn upward which cuts the horizontal axis showing the required airspeed of flight, - the point of intersection giving the speed of 165 km/hr. The set of curves for the wing loading is obtained by solving the equation

$$\frac{A}{F} \quad \frac{c_a}{200} \quad \frac{\gamma}{g} v^2$$

for consecutive values of A/F (e.g., 30 kg/m², etc.) and for determining the related values of c_a and v, when $\frac{\gamma}{g} = \frac{1}{8}$ and by plotting the ascertained speeds v (to a suitable scale) vertically downward, commencing at the horizontal axis of c_a. In this procedure the relations are chosen in such a manner that we can read the speed in km/hr and the values of c_a along the horizontal axis, on the same scale. The straight lines representing the altitudes of flight are so inclined that each of them, in conjunction with the corresponding horizontal part of the lines which may be drawn to connect the various quantities (the line of transference), gives the speed in horizontal flight corresponding to the air density for the particular altitude.

For the airplane to maintain horizontal flight, it is necessary to use the power

$$L = A \frac{c_w}{c_a} v \text{ kgm/sec.}$$

This gives us

$$\frac{\frac{L}{A}}{v} = \frac{c_w}{c_a}$$

If we then extend the vertical transfer line above the c_a axis so as to intersect the $\frac{c_a}{c_w}$ - line (in our example $\frac{c_a}{c_w} = 8$ kg/HP), on which the starting point (here 1.5) of the polar curve lies, then the ordinate, measured from the c_a axis, gives the power required for sustentation in HP/kg. If these values are determined for several angles of attack and for various altitudes (in the example: 0, 2000, 3000 and 4000 meters), then, by joining the points thus obtained for each altitude, a curve is obtained which gives the relation between the HP/kg required for sustentation and the flight speed.

In addition to the comparison with the required power for sustentation, the available propeller output in HP/kg is likewise plotted against the speed of flight. For this purpose, the propeller efficiency η, is plotted in the middle diagram on the left-hand side, against the speed of flight. If no experimental data are available, a maximum value for the efficiency can be assumed; while we may, first of all, determine the theoretical efficiency, on the basis of the outflow theory according to the probable speed, and the load on the disc area of the propeller in HP/sq.m (in the left-hand bottom figure) About 12 to 15% must be deducted from the result. The continuation of the efficiency curve is assumed to have a parabolic form, such that when the speed is zero the efficiency is also zero.

With the same scale as that for the efficiency, the ratio $\frac{N_h}{N_o}$, of the engine B.HP on the ground and at an altitude h, is plotted for a number of different altitudes. In the case under consideration, the values of the ratios for the 260 HP Daimler engine (D IVa) are plotted on the left-hand side and those for the 240 HP Maybach engine (Mb IVa) on the right-hand, and each connected by radiating lines with some convenient point on the axis of the abscissas. If a horizontal line is drawn from any point on the efficiency curve (e.g., from v = 140 km/hr) to the curve, for altitude of 0 m. of the Maybach engine and, from the point of intersection, a vertical is dropped to the curve for an altitude of 3000 m., then the ordinate of the point where the vertical cuts this curve is $\eta = \frac{N_h}{N_o}$ for 3000 m. altitude, as can be easily seen by comparison. This value must now be multiplied by $75 \frac{N_o}{A}$ in order to obtain the output in HP/kg and compare it with the power required for sustentation. For this purpose, we can construct a second scale divided into units of $75 \frac{N_o}{A}$, draw another set of radiating lines from any convenient point on the axis of the abscissas and then proceed as before. If the distance of the origin of the radiating lines from the vertical on which $\frac{N_h}{N_o}$ are plotted corresponds to 270 km/hr = 75 m/sec, from the scale of speeds on the c_a axis, then by analogy:

$$\frac{\frac{L}{A}}{v} = \frac{75 \frac{N_o}{A}}{75} = \frac{N_o}{A}$$

Since further

$$\frac{\frac{L}{A}}{v} = \frac{c_w}{c_a}$$

then

$$\frac{N_o}{A} = \frac{c_w}{c_a}$$

The set of radiating lines for values of $\frac{N_o}{A}$ is, therefore, identical with that for $\frac{c_w}{c_a}$, which can likewise be used for the operations involved in the multiplication of $\eta \frac{N_h}{N_o}$ by $\frac{N_o}{A}$ 75, after inscribing on the radiating lines the reciprocal values of $\frac{N_o}{A}$. This condition has been fulfilled in the diagram. The values of $\frac{N_h}{N_o}$ are plotted on a vertical axis, the distance of which from the origin of the radiating $\frac{c_a}{c_w}$ lines corresponds to a speed of 270 km/hr. The output is determined by proceeding horizontally from the point already found on the line of 3000 m. altitude, till it intersects the dotted $\frac{c_a}{c_w}$ curve on the larger diagram corresponding to N_o, and thence vertically till it meets the dash radial line of $\frac{c_a}{c_w}$, which corresponds to the power loading of the airplane (in the example 12 kg/HP). The ordinate at the point of intersection measured from the c_a axis, gives the power transmitted by the propeller to the airplane, in HP/kg on the same scale as that for the power required for sustentation. Through a horizontal line drawn to the left, we may connect these outputs to the ordinate of the flight speed (in the example, 140 km/hr), which is the speed on the parabolic curve of propeller efficiency from which we started. In this way, we may also draw the curves of propeller output at all altitudes for which the curves giving the power required for sustentation have been obtained.

The vertical distance between the curves for the propeller output at various altitudes and the power required for sustentation at the same altitude, corresponds to $\frac{S-L}{A}$ and is, therefore, proportional to the climbing speed of the airplane when flying at the speed determined by the position of the ordinate along the abscissas. The scale for the climbing speed is on the left-hand side of the diagram. The greatest vertical interval between corresponding curves for the propeller output and the power required for sustentation, gives the position of the maximum climbing speed. The times of climb are obtained by graphic integration, after plotting the climbing speed against the altitude of flight. The points of intersection of the corresponding curves, giving the propeller output and the power required for sustentation, represent the points of maximum and minimum flying speeds. The latter is of no practical importance. The ceiling is given by the associated curves of propeller output and power required for sustentation which just touch each other (in the example at 4000 m.). The angle of attack for each position of flight is given by the direct relation between the polar curves and the curves of power required for sustentation.

In this calculation of climbing speed, it is tacitly assumed that the lift always remains equal to the weight of the airplane. Since, in climbing, $A = G \cos \alpha$ must be taken into consideration, where α is the angle between the flight path and the horizontal, an error creeps into the speed which, however, even with airplanes having a large reserve of power and near the ground, never exceeds

2%, so that the total time of climb is hardly affected.

The results of this graphic calculation agree well with actual performances. Small variations are unavoidable, since the method described is based on an average atmospheric pressure variation corresponding to

$$\frac{\gamma_h}{\gamma_o} = 0.9^H$$

The minimum power required for sustentation is attained at each altitude with the same angle of attack, as can be seen from the following:

$$A = \xi_a \frac{\gamma_h}{g} v^2 F$$

$$L = A \frac{\xi_w}{\xi_a} v$$

On eliminating v, we get

$$L = \sqrt{\frac{A^3}{F} \frac{g}{\gamma_h} \frac{\xi_w^2}{\xi_a^3}}$$

With a given altitude and airplane, the minimum power required for sustentation L, attains its minimum value when $\frac{\xi_w^2}{\xi_a^3}$ or $\frac{c_w^2}{c_a^3}$ has the smallest value. It might be supposed that this would also be the angle of attack at the ceiling, but this is not the case. If the power required for sustentation and the engine output are assumed to be equal, the equation takes the following form:

$$\frac{\xi_w^2}{\xi_a^3 \eta^2} \frac{A}{F} \left(\frac{H}{N_o}\right)^2 \frac{g}{\gamma_o} \frac{1}{(75)^2} = f_1(H)\ f_2(H)^2 = f(H)$$

in which $f(H) = \frac{\gamma_h}{\gamma_o}$ expresses the decrease in air density with altitude and $f_1(H) = \frac{\gamma_h}{\gamma_o}$, the decrease in engine output with altitude, f(H) decreases continuously with increasing altitude. The ceiling is, therefore, reached quite independently of the variations in f(H), when $\frac{\xi_w^2}{\xi_a^3 \eta^2}$ (or the proportional expression $\frac{c_w^2}{c_a^3 \eta^2}$) is a minimum, η and c_a being related to each other through v. At the ceiling, therefore, the airplane flies with an angle of attack which need not always correspond to the minimum value of $\frac{c_w^2}{c_a^3}$.

This is also clearly seen, if the ceiling is found by the graphic method. The curves of propeller output and of the power required for sustentation touch at a point which only coincides with the minimum value of the power required for sustentation, when the highest propeller efficiency is attained at the ceiling speed.

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Weight 11500 kg

11.2 kg/HP; 40 kg/m²

Wing area 287 m²